# Reading Assignment 4 (Due Friday 7/2/21 by 12:55 PM)

**Basic learning objectives:** These are the tasks you should be able to perform with reasonable fluency **when you arrive at our next class meeting**. Important new vocabulary words are indicated in italics.

1. State the definitions of *vector-valued functions*, the *graph* of a vector valued function, and the *parametric equations* of a curve. Describe the relationship between vector-valued functions and parametric equations.
2. State various examples of vector-valued functions and parametric curve.
3. Graph parametric curves using a graphing calculator or other appropriate technology (like GeoGebra or Desmos).
4. Parameterize various curves such as lines and circles and ellipses in $\mathbb{R}^2$ or $\mathbb{R}^3$, the intersection of two graphs, etc.
5. Parameterize the traces and level curves of a function $f(x, y)$ of two variables.

**Advanced learning objectives:** In addition to mastering the basic objectives, here are the tasks you should be able to perform **after class, with sufficient practice:**

1. State the definition of and describe geometrically the derivative $\mathbf{r}'(t)$ of a vector-valued function $\mathbf{r}(t)$. Give a physical interpretation when $\mathbf{r}(t)$ describes displacement or velocity.
2. Compute derivatives of vector-valued functions using both the definition and other techniques. In particular, utilize the differentiation rules to perform more advanced computations.
3. Compute the tangent line to a curve. Compare and contrast the tangent line to a curve with the tangent line to a graph from ordinary calculus. Recognize a tangent line as a linear approximation to a curve.
4. Integrate vector valued functions using antiderivatives. Give a physical interpretation when the vector-valued function you are integrating represents velocity or acceleration.

**Directions:** Read the following sections of the book:

- Sections 9.4.5 and 9.4.6 (no additional tasks - we covered the main ideas in class). Optional: 9.4.4 (9.4.4 won't be on weekly assignments or exams).
- Sections 9.5.3 and 9.5.4.
- All of Section 9.6.
- Section 9.7 up to and including Preview Activity 9.7.1.

and complete the following tasks along the way. If an Activity is not listed, you do not need to complete it (although you are welcome to read it). Turn your write up in via gradescope. You do not need to write the questions down, as long as you clearly indicate the question number.

**1.** Preview Activity 9.6.1.

**2.** Activity 9.6.2. I recommend using GeoGebra to graph the curves. You can find directions online for plotting parametric curves with GeoGebra. You do not need to include the graphs in your write-up.

**3.** Activity 9.6.3. I recommend using GeoGebra or GeoGebra 3D. You do not need to include the graphs in your write-up.

For part (e), you must:

- create a curve as directed using GeoGebra, or some other online graphing calculutor.
- post a link to your graph on the Zulip forum. To pass this assignment, you must post your link **by the due date** on the Zulip:reading 4 task 3 stream using **your name** as the topic.

**4.** Activity 9.6.4

**5.** Find a vector valued function $\mathbf{r}(t)$ that describes the curve at the intersection of the paraboloid $z = 5x^2+5y^2$ and the (parabolic) cylinder $y = 5x^2$. Hint: use $x = t$ as the parameter. Plot your parameterization. You can compare with mine to see how you did: GeoGebra: Intersection of Paraboloid and Cylinder.

**6.** Write down 3 things you learned or still have questions about in Section 9.6. **I will not be lecturing on Section 9.6 in class, but we will do a reading debrief so that you can ask me questions.**

**7.** Preview Activity 9.7.1. Before completing the activity, it would be a good idea to review the derivative of a function $f(x)$. In particular, try to understand why the derivative measures the slope of the tangent line. Here is a visualization of what's going on: GeoGebra: Tangent Line Approximation by Secant Line. Notice that the difference quotient $\frac{f(x+h)+f(x)}{h}$ is the slope of the secant line through the points $(x, f(x))$ and $(x+h, f(x+h))$. As we take $h \to 0$ in the limit, the secant line approaches the tangent line. Thus, $f'(x) = \lim_{h\to 0} \frac{f(x+h)+f(x)}{h}$ is the slope op the tangent line to the graph of $f$ at the point $(x, f(x))$. We will be generalizing this idea to define a derivative of a vector-valued functions.

**8.** We have seen that a vector-valued function $\mathbf{r}(t)$ that describes a line through a point $P$ in the direction of $\mathbf{v}$ is given by $\mathbf{r}(t) = \overrightarrow{OP} + t\mathbf{v}$. Based on the intuition you developed in Preview Activity 9.7.1, what do you think the derivative $\mathbf{r}'(t)$ is? What is its direction relative to the line traced out by $\mathbf{r}(t)$? What does it represent? Explain your reasoning.

**9.** Suppose that $\mathbf{s}(t)$ is a vector-valued function that describes the displacement $\mathbf{s}(t)$ of an object with respect to the origin. Based on your intuition and previous studies of derivatives, what you do think the vector $\mathbf{s}'(t_0)$ at time $t_0$ represents? What is it's direction relative to the curve traced out by $\mathbf{s}(t)$? Explain your reasoning.